

CONCEPT NOTE • HULL CH 11

Put-call parity, and the bounds that fall out of it

A call and a put are the same trade in different packaging. One equation gives you the price relationship, both lower bounds, and the arbitrage when prices drift apart.

THE IDEA

Two portfolios that are worth the same at expiry in every state of the world must cost the same today. A call plus cash is one such pair with a put plus the share. Everything else on this page is that sentence rearranged.

The equation

For European options on a stock that pays no dividend, with the same strike and expiry:

$$c + Ke^{-rT} = p + S_0$$

SYMBOL	MEANING	UNITS
c, p	European call and put prices	\$ per share
S_0	Stock price today	\$ per share
K	Strike price	\$ per share
T	Time to expiry	years
r	Risk-free rate, continuously compounded	per year

Ke^{-rT} is the present value of the strike: the amount you would lend today to have exactly K at expiry. It is the same discount factor used to value a forward contract you already hold.

Why it holds: two portfolios

Portfolio A holds one call and a zero-coupon bond paying K at expiry. **Portfolio C** holds one put and one share. Look at what each is worth on the expiry date, T :

AT EXPIRY	IF $S_T > K$	IF $S_T < K$
A: call	$S_T - K$	0
A: bond	K	K
A: total	S_T	K
C: put	0	$K - S_T$
C: share	S_T	S_T

AT EXPIRY

IF $S_T > K$ IF $S_T < K$

C: total

 S_T K

Both portfolios pay the larger of S_T and K , whatever the stock does. If they cost different amounts today, you could buy the cheap one, sell the dear one, and pocket the difference with nothing owed at expiry. Competition for that free money pushes the two prices together, which is the equation.

The proof needs European options. An American option can be exercised early, so the two portfolios are no longer guaranteed to be held to the same date.

REAL MARKET DATA

Netflix, quotes at the close on Monday, September 21, 2026 (CBOE, 15-minute delayed), with time measured from the next morning. Netflix pays no dividend, so parity applies exactly.

INPUT	VALUE
Stock, S_0	73.43
Expiry	March 19, 2027 · 178 days from September 22 · $T = 0.4877$
Rate, r	4.1957% continuously compounded (6-month Treasury at 4.24%)
Discount factor, e^{-rT}	0.979747
March 75 call, mid of 7.90 / 8.15	8.03

Solve for the put: $p = c + Ke^{-rT} - S_0 = 8.03 + 75 \times 0.979747 - 73.43 = 8.03 + 73.48 - 73.43 = 8.08$

The market's March 75 put was quoted 8.05 / 8.20, a mid of 8.12. Parity was off by four cents, inside the bid-ask spread.

Rearranging it: synthetic positions

Move terms across and each rearrangement is a trade you can build:

- $p = c - S_0 + Ke^{-rT}$. A **synthetic put** is: buy the call, short the share, and lend the present value of the strike. The lending leg is the one people forget.
- $c = p + S_0 - Ke^{-rT}$. A **synthetic call** is: buy the put, buy the share, and borrow the present value of the strike.
- $c - p = S_0 - Ke^{-rT}$. Long a call and short a put at the same strike is a **forward position**. Its payoff is a straight line through K with no kink.

Most option strategies are this equation with terms moved around.

Both lower bounds, one line each

Start from $c - p = S_0 - Ke^{-rT}$. An option can never be worth less than zero.

- Set $p = 0$ and the call is at least $S_0 - Ke^{-rT}$.
- Set $c = 0$ and the put is at least $Ke^{-rT} - S_0$.

Neither price can be negative, so:

$$c \geq \max(S_0 - Ke^{-rT}, 0) \quad p \geq \max(Ke^{-rT} - S_0, 0)$$

You do not need to memorize these. Write parity and delete a letter.

REAL MARKET DATA

The bounds on the same Netflix chain ($S_0 = 73.43$, $e^{-rT} = 0.979747$)

	CALCULATION	RESULT
March 65 call, lower bound	$73.43 - 65 \times 0.979747$	9.75
March 65 call, intrinsic value	$73.43 - 65$	8.43
March 65 call, market mid (13.30 / 14.00)		13.65
March 80 put, lower bound	$80 \times 0.979747 - 73.43$	4.95

The call's lower bound sits above its intrinsic value. The extra 1.32 is the interest you save by paying the strike later rather than now, and it is why you would rather sell a call on a non-dividend stock than exercise it early. The market price sits well above the bound because the bound ignores volatility entirely.

When parity breaks, there is a trade

Suppose the screen had shown the March 75 put at 6.59 instead of 8.12. Value both sides:

- Call plus cash: $8.03 + 73.48 = 81.51$
- Put plus share: $6.59 + 73.43 = 80.02$

The put-plus-share side is cheap. Buy it and sell the other side:

TODAY	CASH FLOW	
Buy the put		-6.59
Buy the share		-73.43
Sell the call		+8.03
Borrow Ke^{-rT}		+73.48
Net today		+1.49
AT EXPIRY	IF $S_T = 90$	IF $S_T = 60$
Put	0	+15
Share	+90	+60
Call (assigned when in the money)	-15	0
Repay the loan	-75	-75
Net at expiry	0	0

You collect 1.49 today and owe nothing later, whatever Netflix does. Nobody had to tell you which leg was mispriced: the equation did, and the table proves the profit is riskless. Desks call this trade a **conversion**, and the reverse trade a **reversal**.

Why the money is rarely there

In practice you buy at the ask and sell at the bid. With the real quotes, building a synthetic share by buying the call at 8.15 and selling the put at 8.05 costs $0.10 + 73.48 = 73.58$, which is 0.15 more than simply buying the share at 73.43. Reverse the trade and it is 0.25 worse. Parity held to a few cents, and a few cents is less than the spread you cross to collect it.

What keeps a real-world gap from closing all the way to zero:

- bid-ask spreads on every leg;
- the cost of borrowing shares to sell them short;
- dividends, which change the equation (the share side loses the present value of the dividend);
- American exercise, which breaks the two-portfolio proof.

CHECK YOURSELF

1. A stock trades at 50 and pays no dividend. A six-month European call with strike 50 costs 4.00, and the risk-free rate is 5% continuously compounded. What should the six-month 50 put cost?
2. On the same stock, what is the lower bound for a six-month European call with strike 45?
3. The 50 put in question 1 is quoted at 2.20. Which trade captures the mispricing, and how much does it make today?

ANSWERS

1. $Ke^{-rT} = 50e^{-0.025} = 48.77$, so $p = 4.00 + 48.77 - 50 = 2.77$.
2. $50 - 45e^{-0.025} = 50 - 43.89 = 6.11$.
3. The put is cheap (2.20 against 2.77). Buy the put, buy the share, sell the call, and borrow 48.77: $-2.20 - 50 + 4.00 + 48.77 = +0.57$ today, with zero owed at expiry.