

CONCEPT NOTE • HULL CH 3

Hedging with futures, and how much to hedge

A hedge does not remove risk. It swaps a large price risk for a smaller basis risk, and three formulas tell you which side to take, what price you end up with, and how many contracts to trade.

THE IDEA

Take the futures position that makes money in exactly the move that hurts you. Because you get out before delivery, or the contract is on something slightly different, what you lock in is the futures price plus the basis on the day you close, not the futures price. When the contract is not your asset, the minimum-variance hedge ratio says how much of it to trade.

Which side: ask which way the price has to move to hurt you

- A **short hedge** sells futures. It suits anyone who owns the asset or will sell it later, such as a farmer with a crop in the ground: they lose if the price falls.
- A **long hedge** buys futures. It suits anyone who will buy the asset later, such as an airline that needs fuel: they lose if the price rises.

Do not memorize “producers are short.” Work out your exposure first and the side follows.

Basis: what the hedge actually locks in

The **basis** is the spot price minus the futures price, $b = S - F$, both in \$ per unit. At delivery the futures price converges to spot, so the basis is zero. On any earlier day it is not, and nobody knows in advance what it will be.

Say you put on a short hedge at futures price F_1 and close it at time 2, when spot is S_2 and futures is F_2 . You sell the asset for S_2 and gain $F_1 - F_2$ on the futures. Your **effective price** is

$$S_2 + (F_1 - F_2) = F_1 + (S_2 - F_2) = F_1 + b_2$$

F_1 is known when you put the hedge on. The only uncertain piece is b_2 , the basis on the day you get out. That is **basis risk**, and it is what is left after hedging. A long hedger pays $F_1 + b_2$ by the same arithmetic.

WORKED EXAMPLE

A wheat farmer will sell 100,000 bushels in mid-November and shorts 20 December contracts (5,000 bushels each) at $F_1 = 5.80$. In November she sells the grain at $S_2 = 5.35$ and buys back the futures at $F_2 = 5.52$, so $b_2 = 5.35 - 5.52 = -0.17$.

She receives 5.35 for the grain plus a futures gain of $5.80 - 5.52 = 0.28$, so 5.63 a bushel (\$563,000 in total). Check: $F_1 + b_2 = 5.80 - 0.17 = 5.63$. Spot fell 45 cents, and she still received 5.63 rather than 5.35. She did not get 5.80 either: the 17-cent gap is the basis on the day she closed.

If the basis rises (becomes less negative or more positive) it has **strengthened**; if it falls it has **weakened**. A short hedger receives $F_1 + b_2$, so a stronger basis helps the short and hurts the long. Read the sign from the formula rather than memorizing which is “good.”

Basis risk comes from two mismatches. **Timing**: you close before delivery, so convergence has not finished. **Asset**: the contract is on something close to, but not the same as, what you own. To keep the timing mismatch small, choose the contract month closest to, but **after**, the end of the hedge. A contract that expires before your exposure ends leaves you unhedged for the last stretch. Liquidity points the same way, since far months trade thinly.

When the contract is not your asset: the hedge ratio

There is no futures contract on jet fuel. An airline hedges with heating oil or crude, which move with jet fuel but not one for one. Trading one unit of futures per unit of exposure is then not the best you can do.

Hold one unit of the asset and short h units of futures. Over the hedge the position changes by $\Delta S - h \Delta F$ and its variance is $\sigma_S^2 + h^2 \sigma_F^2 - 2h\rho\sigma_S\sigma_F$. Set the derivative with respect to h to zero:

$$h^* = \rho \frac{\sigma_S}{\sigma_F} \quad N^* = h^* \frac{Q_A}{Q_F}$$

Here σ_S and σ_F are the standard deviations of changes in the spot and futures prices (\$ per unit), ρ is the correlation between those changes, Q_A is the size of your exposure and Q_F the size of one contract (units), and N^* is the number of contracts.

REAL MARKET DATA

Jet fuel against two candidate contracts, weekly price changes, October 10, 2025 to October 2, 2026.

Spot: US Gulf Coast jet fuel (EIA, via FRED series DJFUELUSGULF). Futures: front-month NYMEX heating oil and WTI crude (Friday closes via Yahoo Finance; crude at 42 gallons a barrel). Weeks that span a contract roll are dropped, since the jump to the next contract is not a price change anyone earned, leaving 25.

	HEATING OIL	WTI CRUDE
σ_S , jet fuel (\$/gal)	0.3163	0.3163
σ_F , futures (\$/gal)	0.2949	0.1578
ρ	0.9147	0.7791
h^*	0.9811	1.5617
Share of variance removed, ρ^2	0.837	0.607

An airline buying 5,000,000 gallons hedges with long positions. Both contracts cover 42,000 gallons, so $Q_A/Q_F = 119.05$:

- Heating oil: $N^* = 0.9811 \times 119.05 = 116.8$, so **long 117** contracts.
- Crude: $N^* = 1.5617 \times 119.05 = 185.9$, so **long 186** contracts.

σ_S/σ_F sizes the hedge: if your asset moves twice as much as the contract, you need twice as many. ρ shrinks it: a weaker link means fewer contracts, because past a point you are no longer hedging, you are taking a view on heating oil. h^* is also the slope of a regression of ΔS on ΔF , and ρ^2 , the regression's R^2 , is the share of variance the hedge removes. That share is called **hedge effectiveness**.

Heating oil is the better hedge: it removes 84% of the variance against crude's 61%. The crude ratio is above 1 because jet fuel moved more in dollars per gallon than crude did: a hedge ratio is not capped at 1. The estimate also depends on the interval. Measured in *daily* changes, the same year gives a heating-oil ratio of 0.81 and an effectiveness of 63%, because day-to-day timing noise between the two markets washes out over a week. Estimate over an interval close to your hedge horizon.

NOTE

Tailing the hedge. Because futures settle daily, practitioners often size the hedge in dollars: estimate $\hat{h}$ from daily *percentage* changes and trade $N^* = \hat{h} V_A/V_F$, with V_A the dollar value of the exposure and V_F the dollar value of one contract (futures price times size), adjusting the count as prices move. The difference from the simple formula is usually small.

Hedging a stock portfolio: beta is the hedge ratio

A stock portfolio is a cross hedge too: there is no contract on your 40 stocks, only on an index. The portfolio's **beta**, β , measures how much its return moves with the index return, so it plays the role of h^* in percentage terms. With V_A the portfolio value, F the index futures price and m the contract multiplier (\$ per index point), the number of contracts that moves you from beta β to a target β^* is

$$N = (\beta^* - \beta) \frac{V_A}{F \times m}$$

A negative N means sell. The sign comes from $(\beta^* - \beta)$, not from the word “hedge”: index futures are a dial on market exposure, and a full hedge ($\beta^* = 0$) is one setting.

REAL MARKET DATA

\$10 million in the Technology Select Sector SPDR fund (XLK), hedged with December 2026 E-mini S&P 500 futures (ESZ6) at the close of Thursday, October 8, 2026 (CME, via Yahoo Finance). Beta of 1.72 estimated from 51 weekly price returns of XLK against the S&P 500, October 10, 2025 to October 2, 2026. The multiplier is \$50, so one contract is worth $7,816.25 \times 50 = \$390,812.50$ and $V_A/(F \times m) = 25.59$.

TARGET BETA	CALCULATION	CONTRACTS
0 (fully hedged)	$(0 - 1.72) \times 25.59$	short 44
1.00 (market risk only)	$(1.00 - 1.72) \times 25.59$	short 18
2.00 (more exposure)	$(2.00 - 1.72) \times 25.59$	long 7

Tie-out for the full hedge: if the index falls 5%, the portfolio is expected to lose $1.72 \times 5\% \times 10,000,000 = \$860,000$, and 44 short contracts gain $44 \times 50 \times (7,816.25 \times 5\%) = \$859,788$.

Why hedge instead of selling the stocks? Selling forty positions and buying them back costs commissions and may trigger tax. The real reason is that you hold them because you think they will beat the market. A beta hedge removes the market and keeps that: what is left is roughly the risk-free rate plus your excess

return over the market, which can of course be negative.

Where hedges go wrong

- **Rolling.** If your exposure runs longer than the liquid contracts, you hedge with a near contract and roll it forward again and again. Each roll adds its own basis risk.
- **Cash before profit.** In 1993 Metallgesellschaft's US subsidiary sold customers years of oil products at fixed prices and hedged with rolled short-dated futures. When oil fell, the margin calls on the futures arrived at once, while the offsetting gain on the customer contracts was spread over years. The parent closed the position with losses of more than a billion dollars. The hedge was defensible on paper; the cash flows broke it.
- **Not everyone should hedge.** Shareholders can diversify on their own. A firm that hedges when its competitors do not can end up more exposed, because its selling prices still move with costs. And a hedge that loses money on the futures side is hard to explain, even when it did exactly its job. Hedging reduces variance, not regret.

CHECK YOURSELF

1. A bakery will buy wheat in March and goes long March futures at $F_1 = 6.10$ when spot is 6.02. When it buys, spot is 6.40 and futures are 6.52. What effective price does it pay, and did the basis strengthen or weaken?
2. A utility will buy 600,000 MMBtu of gas at a regional hub and hedges with Henry Hub futures (10,000 MMBtu each). $\sigma_S = 0.36$, $\sigma_F = 0.30$, $\rho = 0.85$. How many contracts, and which side? What if ρ were 0.60?
3. A \$25 million portfolio has beta 0.85. An index futures contract is at 7,900 with a \$50 multiplier. How many contracts take the beta to 0.40, and to 1.20?

ANSWERS

1. $b_2 = 6.40 - 6.52 = -0.12$. Effective price = $F_1 + b_2 = 6.10 - 0.12 = 5.98$ (spot 6.40 less a futures gain of 0.42). The basis went from -0.08 to -0.12 : it weakened, which helps a long hedger.
2. $h^* = 0.85 \times 0.36/0.30 = 1.02$ and $N^* = 1.02 \times 600,000/10,000 = 61.2$, so long 61. With $\rho = 0.60$: $h^* = 0.72$, $N^* = 43.2$, so long 43. A weaker link means a smaller hedge, not a bigger one.
3. One contract is $7,900 \times 50 = 395,000$, and $25,000,000/395,000 = 63.29$. To 0.40: $(0.40 - 0.85) \times 63.29 = -28.5$, short 28 or 29. To 1.20: $(1.20 - 0.85) \times 63.29 = +22.2$, long 22.