

CONCEPT NOTE • HULL CH 5

Where the forward price comes from

A forward price is not a forecast. It is the cost of buying the asset today and carrying it to the delivery date, enforced by two trades. This note builds the price, values a contract you already hold, and shows where the argument breaks.

THE IDEA

If you can buy an asset today and hold it until the delivery date, then agreeing to buy it later must cost the same as buying it now and paying to carry it. Carry is the interest you pay to hold the asset, minus any income it pays you while you wait. A forward quoted at any other price hands someone a riskless profit.

The price is a trade

Take gold, which investors hold as a store of value and which pays nothing while you hold it (storage is small for gold, so ignore it for now). Let S_0 be the spot price today and F_0 the forward price agreed today for delivery in T years, both in dollars per ounce, and let r be the risk-free rate, continuously compounded, per year.

There are two ways to own an ounce on the delivery date. You can agree today to buy it then at F_0 . Or you can borrow S_0 , buy the ounce now and hold it, which leaves you owing $S_0 e^{rT}$ at delivery. Both end with one ounce on the same date, so they must cost the same:

$$F_0 = S_0 e^{rT}$$

WORKED EXAMPLE

Gold at 4,200 an ounce, $r = 4\%$, one year. The fair forward is $4,200e^{0.04} = 4,371.41$. Suppose it is quoted at 4,450, too high. Buy the gold with borrowed money and sell it forward (a **cash-and-carry** trade):

LEG	TODAY	IN ONE YEAR
Borrow 4,200	+4,200.00	-4,371.41
Buy one ounce	-4,200.00	
Sell the forward at 4,450, deliver the ounce	0	+4,450.00
Net	0	+78.59

If the forward is too low, run the trade backwards. That needs **short selling**: you borrow the asset from someone who owns it, sell it, and hand it back later, paying the lender any income it would have earned in the meantime.

WORKED EXAMPLE

Same gold, forward quoted at 4,300, too low. A reverse cash-and-carry:

LEG	TODAY	IN ONE YEAR
Borrow one ounce and sell it	+4,200.00	
Lend the 4,200	-4,200.00	+4,371.41
Buy the forward at 4,300, return the ounce	0	-4,300.00
Net	0	+71.41

Neither trade needs a view on gold. The first pushes an expensive forward down, the second pushes a cheap one up, and the only price neither can attack is $S_0 e^{rT}$. In practice the cheap side is usually closed by people who already own gold: they sell their ounce, lend the cash and buy the forward, which is the same trade without the borrowing. Keep the short-selling step in mind, because it is where the argument breaks later.

Income pulls the forward down

If the asset pays you something while you hold it, the person carrying it collects that income, so the forward must be lower by exactly that amount. Two ways to write it:

$$F_0 = (S_0 - I)e^{rT} \quad F_0 = S_0 e^{(r-q)T}$$

I is the present value **today** of every cash payment the asset makes before delivery (\$ per unit). q is the same income expressed as a continuous yield (per year). Discount each payment back to today before you subtract it; subtracting the raw dividends is the most common slip.

WORKED EXAMPLE

A stock at 80, nine-month forward, $r = 5\%$, dividends of 1.20 in two months and five months.

$$I = 1.20e^{-0.05 \times 2/12} + 1.20e^{-0.05 \times 5/12} = 1.1900 + 1.1753 = 2.3653$$

$$F_0 = (80 - 2.3653)e^{0.05 \times 0.75} = 80.60, \text{ against } 80e^{0.0375} = 83.06 \text{ with no dividends.}$$

The yield version covers more than stocks. For a **stock index**, q is the index's dividend yield. For a **currency**, holding the foreign money earns the foreign risk-free rate r_f , so it plays the role of q :

$$F_0 = S_0 e^{(r-r_f)T}$$

with S_0 in dollars per unit of foreign currency. If the foreign rate is higher than the dollar rate, the forward sits below spot. It has to: otherwise you would borrow dollars, convert, earn the higher rate and sell the currency forward at no discount.

Price is not value

A forward is worth zero the day it is struck, because nobody pays anything to enter it. After that the market's forward price F_0 moves and your delivery price K does not. Compare your contract with a new one struck today for the same date: the only difference at delivery is $F_0 - K$ in cash, a number you already know. Discount it.

$$f_{\text{long}} = (F_0 - K)e^{-rT} \quad f_{\text{short}} = (K - F_0)e^{-rT}$$

Here T is the time *remaining*. Suppose you bought the gold forward above at its fair price, $K = 4,371.41$. Six months later gold has fallen and the six-month forward is 4,250, with r still 4%. Your contract is worth $(4,250 - 4,371.41)e^{-0.02} = -119.01$ an ounce. You did nothing wrong: a fair price is fair on the day you strike it, and after that it is just a position. (Which r ? Strictly, the rate your counterparty pays on the margin you post, typically SOFR or fed funds. Over a few months any short-term risk-free rate does the job.)

REAL MARKET DATA

Three futures curves, settlement prices from Wednesday, October 7, 2026 (COMEX gold, CME euro FX, NYMEX WTI crude, via Yahoo Finance; Treasury yields from FRED, same day). Each pair of contracts is one year apart, so $\ln(F_{\text{Dec 27}}/F_{\text{Dec 26}})$ is the net carry per year that the market priced into 2027.

ASSET	DEC 2026	DEC 2027	NET CARRY, % PER YEAR
Gold, \$ per ounce	4,140.70	4,356.10	5.07
Euro, \$ per euro	1.12300	1.14145	1.63
WTI crude, \$ per barrel	87.55	76.42	-13.60

- **Gold** pays nothing and is cheap to store, so its carry should be close to the cost of borrowing dollars through 2027. The 1-year Treasury yield was 4.42% and the 2-year 4.77%, which together price one-year money a year out at about 5.1% (continuously compounded). The curve slopes up because of interest, not because anyone expects gold to rise: a futures price is not a forecast.
- **The euro** earns the euro interest rate. Its curve rose 1.63% a year, the gap the market expected between dollar and euro rates over 2027. Overnight that day, dollars paid 3.88% (SOFR) and euros 2.44% (€STR).
- **Crude** fell 13.60% a year. Interest and storage both push a forward *up*, so nothing in the formulas above can produce a falling curve. That is the next section.

When the argument breaks

The argument assumed you can borrow and lend at one rate, short the asset freely, and trade at no cost. Each exception below is one of those failing.

Consumption assets. Oil, copper and wheat are held to be used, not to be invested. Storage is a cost, which acts like negative income: $F_0 = (S_0 + U)e^{rT}$, where U is the present value of storage, or $S_0e^{(r+u)T}$ with u as a yield. The cash-and-carry trade still works on oil (borrow, buy a barrel, store it, sell the

forward), so the forward cannot sit *above* full carry. The reverse trade needs you to borrow a barrel and sell it, and a refinery holding oil to keep a plant running will not lend it. Nothing pushes a cheap forward back up, and the equation becomes an inequality:

$$F_0 \leq S_0 e^{(r+u)T}$$

Convenience yield is the name for the gap. Write $F_0 = S_0 e^{(r+u-y)T}$, where y is the benefit of holding the physical barrel rather than a promise of one: the plant keeps running through an outage or a supply shock. Nobody quotes y ; you back it out of prices. On the crude curve above, $\ln(76.42/87.55) = -0.1360 = r + u - y$, so $y - u = r + 13.60\%$. With r between 4.4% and 5.1%, the convenience yield net of storage was about 18% to 19% a year. The market was paying heavily for barrels now over barrels in a year, which is what a backwardated curve is.

Borrowing costs more than lending. If you borrow at r_b and lend at a lower r_l , each trade is enforced at a different rate, and the forward can sit anywhere in a band: $S_0 e^{r_l T} \leq F_0 \leq S_0 e^{r_b T}$. Bid-ask spreads and the fee for borrowing the asset widen the band further. Inside it, no trade pays.

Futures versus forwards. A future settles gains and losses daily. With known interest rates the two prices are equal. If the asset tends to rise when rates rise, a long future reinvests its gains at high rates and funds its losses at low ones, so the futures price sits slightly above the forward (below, if they move oppositely). The gap is small, and practice usually ignores it, as this note does.

CHECK YOURSELF

1. A stock trades at 120 and will pay a dividend of 2.00 in four months. The risk-free rate is 4.5% continuously compounded. What is the six-month forward price?
2. The same six-month forward is quoted at 125. Which trade do you put on, and what do you make per share at delivery?
3. You are long a forward to buy 1,000,000 euros at 1.1000 dollars per euro. Three months remain, the three-month forward is now 1.1250, and $r = 4\%$. What is your contract worth?
4. Crude trades at 70 and the one-year future at 66. The risk-free rate is 4% and storage costs 2% a year, both continuously compounded. What convenience yield does the market imply?

ANSWERS

1. $I = 2.00e^{-0.045 \times 4/12} = 1.9702$, so $F_0 = (120 - 1.9702)e^{0.045 \times 0.5} = 120.72$.
2. The forward is too high. Borrow 120, buy the share and sell the forward at 125. At delivery you receive 125, repay $120e^{0.0225} = 122.73$, and still hold the dividend reinvested for two months, $2.00e^{0.045 \times 2/12} = 2.02$. Profit $125 - 122.73 + 2.02 = 4.28$, which is $125 - 120.72$.
3. $(1.1250 - 1.1000) \times 1,000,000 \times e^{-0.04 \times 0.25} = 25,000 \times 0.990050 = 24,751.25$ dollars.
4. $\ln(66/70) = -0.0588 = 0.04 + 0.02 - y$, so $y = 0.1188$, about 11.9% a year.